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Quantifying the Aesthetic Complexity of Tamahagane Structural Colors by Persistent Homology

Hideyuki Kanematsu[a,b]*, Akiko Ogawa[c], Yoshiyuki Matsui[d], Michiko Yoshitake[e], Takeshi Bessho[e], Ryoichi Ichino[e], Yoshitaka Adachi[f], Takayoshi Nakano[a]

[a]*Div. Materials & Manufacturing Science, Graduate School of Engineering, The University of Osaka, 2-1, Yamadaoka, Suita, Osaka 565-0871, Japan*

[b]*BEL Inc.2-4-31, Shinonome Nishi-machi, Sakai-ku, Sakai, Osaka, 590-0013,Japan*

[c]*Dept. Chemistry & Biochemistry, National Institute of Technology, Suzuka College, Shiroko-cho, Suzuka, Mie 510-0294, Japan*

[d]*Graduate School of Arts and Design, Kobe Design University, 8-1-1 Gakuennishi-machi, Nishi-ku, Kobe, 651-2196, Japan*

[e]*Institutes of Innovation for Future Society, Nagoya University, NIC, Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601, Japan*

[f]*Department of Materials Design Innovation Engineering, Graduate School of Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8603, Japan*

Abstract

Tamahagane, a traditional Japanese steel produced by the *tatara* process, exhibits multi-colored structural colors described as *gōka-kenran* (gorgeous splendor), whereas lance metal from a modern test converter shows subdued coloration. This contrast has long been recognized but evaluated only subjectively. We applied persistent homology to optical micrographs of both materials, constructing sublevel set filtrations on the hue and saturation channels of HSV-converted images. The β_1 total persistence (enclosed color boundaries) for tamahagane was 1.77 times that of the converter lance metal, and the saturation β_0 mean persistence was 2.08 times greater. These results provide the first reproducible numerical descriptors of the *gōka-kenran* quality, suggesting the applicability of persistent homology to materials archaeology and the quantitative assessment of aesthetic complexity.

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Keywords: tamahagane; persistent homology; structural color; topological data analysis; sublevel set filtration; *tatara* ironmaking

* Corresponding author. Tel/fax.: +81-72-233-1717.
E-mail address: h.kanematsu@mat.eng.osaka-u.ac.jp

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1. Introduction

The Japanese sword has long served as a weapon, yet it has simultaneously been held in the highest regard as a work of art and craftsmanship. Its value derives not merely from its form or historical significance, but is considered to be intrinsic to the structure of the material itself.

Specifically, the Japanese sword possesses a tough internal microstructure while forming a hard, carbon-rich surface layer, thereby achieving the seemingly contradictory properties of hardness and flexibility at once. Such a structure is rational from the standpoint of functional materials design, and may also be interpreted as an embodiment of the concept of harmony characteristic of Japanese culture.



Fig. 1. Japanese traditional swords (*Nihonto*). These swords were originally presented to the late Shoji Kanematsu, father of the corresponding author, upon his retirement, and have since been preserved as family heirlooms.

The steel used as the raw material for the Japanese sword (Fig. 1) is produced by *tatara* ironmaking, a traditional smelting process employing iron sand and charcoal. This process is fundamentally different from modern ironmaking based on the blast furnace–converter route; steel is produced through non-equilibrium and complex reaction pathways. The resulting tamahagane is a distinctive material that is both high in purity and inherently heterogeneous and diverse in microstructure. Its surface, in particular, exhibits a rich palette of colors—gold, blue, purple, and others—giving rise to the appearance described as *gōka-kenran* (gorgeous splendor). This coloration is believed to originate from structural color caused by nanoscale oxide films, yet its complexity has not necessarily been described in quantitative terms.

Meanwhile, recent studies have reported that metal deposits exhibiting coloration similar to that of tamahagane can also form under certain conditions in modern steelmaking processes. This finding suggests that phenomena previously considered unique to traditional ironmaking may be situated within a more universal framework of materials formation. Nevertheless, the differences between tamahagane obtained by traditional *tatara* ironmaking and analogous materials produced by modern processes have thus far been discussed primarily on the basis of empirical and sensory judgment, and a framework for quantitatively evaluating these differences has not been adequately established.

One of the present authors, Matsui, has carried out a comparative study of traditional tamahagane and tamahagane-like lance metal formed during the blowing process in a test converter, focusing principally on the coloration phenomena[1]. It has been suggested that these colors arise from structural color attributable to nanoscale oxide film structures; however, no metric for objectively describing the complexity and spatial distribution of these colors has been proposed to date.

In the present study, we apply persistent homology[2,3], a method of topological data analysis, to optical micrographs of these material surfaces, and attempt to describe the complexity of the color structure in quantitative terms. This approach is expected to enable the beauty of tamahagane, which has hitherto been discussed only in subjective terms, to be captured as a topological quantity. Furthermore, by comparing the morphological features of materials obtained by traditional and modern processes, we demonstrate that persistent homology can offer new insight into the essential nature of tamahagane as a material. In other words, this method has the potential to contribute to the interpretation of a material's history and formation process, and may be regarded as a technique with applicability extending to materials archaeology.

2. Experimental

2.1 Specimens

The materials analyzed in the present study are a tamahagane specimen and a steel sample obtained from a test converter, both provided by one of the authors, Matsui. These specimens are described in detail in Matsui's paper submitted to the Iron and Steel Institute of Japan (ISIJ): Y. Matsui, "Process Engineering Approach to the Origin of High-purity Iron (So-called Tamahagane)," *Tetsu-to-Hagané*, in press (DOI: https://doi.org/10.2355/tetsutohagane.TETSU-2025-094) [1]. Due to copyright restrictions, the original photographs cannot be reproduced in the present paper. Although schematic transformation inevitably modifies fine pixel-level details, the principal spatial organization and color-domain relationships of the original images were intentionally preserved during the stylization process. Persistent homology primarily evaluates topological properties such as connectivity and loop structures across spatial scales rather than exact pixel-level fidelity. Therefore, the PH analysis employed in the present study is considered relatively robust against moderate schematic abstraction, provided that the dominant spatial features and structural color distributions are maintained. Since the principal objective of the present work was to investigate large-scale topological complexity rather than microscopic photometric precision, the schematic representations were considered suitable for exploratory PH analysis.

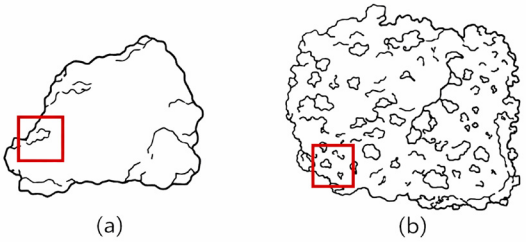

Fig.2 Schematic representation of (a) tradtional tamahagane and (b) converter lance metal.

Fig. 2 shows schematic representations of the overall appearance of (a) tamahagane and (b) converter lance metal, generated by applying AI-based stylization to the original micrographs. Both specimens exhibit regions of multi-colored structural coloration; attention was directed to these colored regions, and enlarged views are presented as schematic illustrations in Fig. 3. Persistent homology analysis was applied to the two images shown in Fig. 3, namely (a) tamahagane and (b) converter lance metal.

The optical micrographs used as the basis for the schematic representations and PH analysis were obtained under identical imaging conditions, including the same microscope setup, illumination source, illumination geometry, camera settings, and magnification. No polarization filter was used during image acquisition. Although the absolute hue values of structural colors may depend on illumination angle and light-source characteristics, the present comparison was performed under the same optical conditions for all samples. Therefore, the observed differences in PH features are considered to reflect relative differences in the spatial organization of the structural color patterns rather than differences caused by imaging conditions.

2.2 Overview of the Persistent Homology Algorithm

The persistent homology analysis employed a Union-Find-based sublevel set filtration combined with scalar feature regression. This model is the most fundamental approach routinely used by the present authors for quantifying image and peak data [6], in which the results of persistent homology are treated as scalar statistical quantities. Specifically, a sublevel set filtration is constructed from the image data, and the Union-Find algorithm is used to efficiently track the birth and death of connected components (H_0). From the resulting persistence diagram, scalar features such as total persistence, mean lifetime, maximum lifetime, and birth/death distributions are extracted. These features serve as explanatory variables for analyzing the image through sigmoid functions or regression models (e.g., nonlinear regression, random forest).

The analysis was carried out in three stages. Stage I involved the conversion of the image to HSV color space. Stage II consisted of sublevel set filtration, and Stage III was the construction of the persistence diagram. While Stages II and III were performed through automated computational procedures, the most critical aspect of the analysis lies in the assumptions adopted in Stage I. These are discussed in detail below.

(i) Linear approximation of the circular nature of hue (the most important assumption)

Hue is inherently a circular quantity defined on the interval 0°–360°. That is, red appears near both 0° and 360°, meaning that H = 5° and H = 355° represent nearly identical colors. However, sublevel set filtration operates by progressively raising a threshold from low to high values over a scalar field, so that hue was treated as a linear scalar mapped to the interval [0, 1]. As a consequence, colors that are perceptually close (e.g., H = 0.01 and H = 0.99, both red) are treated as maximally distant in the filtration. In regions dominated by reddish tones, features that should constitute a single connected component may appear separately at the beginning and end of the filtration, potentially generating spurious features in β_0 and β_1. In the present analysis, the same assumption was applied to both the tamahagane and the converter lance metal specimens, so the validity of the relative comparison is preserved. Nevertheless, a more rigorous treatment could employ circular persistent homology, or alternatively, the analysis could be repeated with multiple hue origin offsets (e.g., 0°, 120°, and 240°) to verify the robustness of the results.

(ii) Image resizing (512 × 384 pixels)

The original images were resized to reduce computational cost. This operation averages sub-pixel color information and may result in the loss of fine-scale spatial color distributions. However, the structural color patterns discussed in Matsui's paper pertain to the micrometer scale and above. Given the 1000 μm scale bar, a width of 512 pixels corresponds to a resolution of approximately 2 μm/pixel, which was judged sufficient for discussing macroscopic color distributions.

(iii) Selection of the HSV color space

Several color spaces are available, including RGB, HSV, Lab, and LCH. In the present analysis, HSV [5] was chosen because the hue channel directly corresponds to the perceived color of the structural coloration—that is, "what color it appears to be." While the *a* and *b* channels of Lab or the chroma (C) channel of LCH are also candidates, hue is the most intuitive parameter for human perception of polychromaticity. In addition, the saturation (S) channel of HSV was included as a supplementary analysis channel, as saturation is related to the thickness and uniformity of the oxide film. A known limitation of HSV is that hue becomes unstable at low values of V (brightness), where small differences in RGB values can produce large fluctuations in hue. Since the images analyzed contain dark background regions, masking based on a threshold value of V should ideally have been applied. This is recognized as an area for improvement in future work.

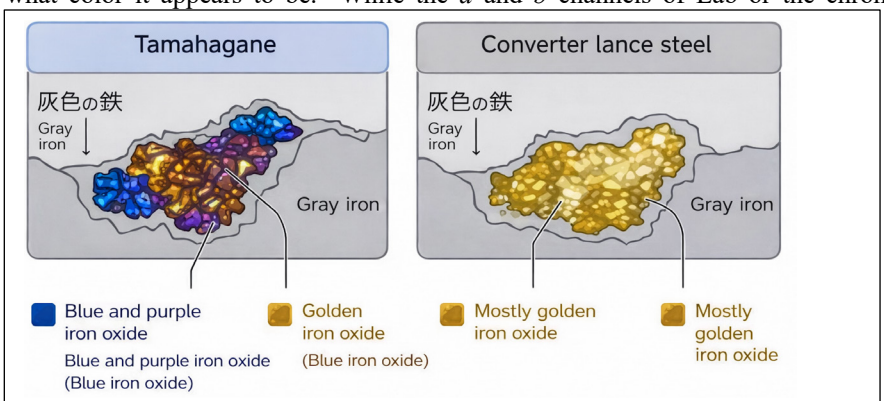

Fig.3 Features representing the luxurious Tamahagane obtained from Fig.2.

(iv) Treatment as a two-dimensional scalar field

The GUDHI Cubical Complex [4] treats the hue channel as a height function defined on a two-dimensional grid. This approach presupposes spatial adjacency between pixels and is consistent with the objective of capturing the spatial arrangement of colors. It should be noted that even if two images share the same color distribution, their persistence diagrams will differ if the spatial arrangement of colors is different. This distinction from simple histogram comparison is precisely the fundamental advantage—and indeed the raison d'être—of applying persistent homology.

The analysis was carried out under these assumptions and conditions. The analysis was implemented in Python 3 using the GUDHI library [4] for cubical complex construction and sublevel set filtration. The complete source code is available from the corresponding author upon reasonable request.

3. Results of Persistent Homology Analysis

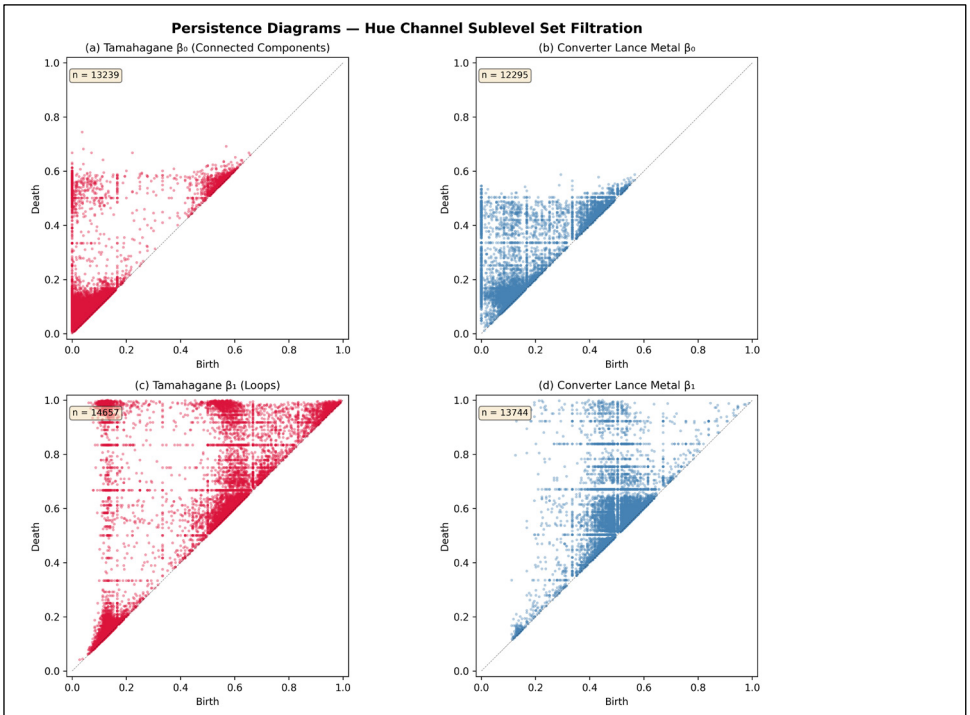

Fig.4 Persistent Diagrams for Tamahagane and converter lance metal.

The results of the persistent homology (PH) analysis are shown in Fig. 4. The persistence diagrams were obtained by applying sublevel set filtration to the hue channel of the HSV-converted optical micrographs. In each diagram, every point represents a topological feature: its horizontal coordinate indicates the filtration value at which the feature is born, whereas its vertical coordinate indicates the value at which it disappears. Points located far from the diagonal correspond to long-lived topological features and are therefore considered structurally significant within the color distribution.

From a materials-science perspective, long-lived H0 features may correspond to spatially separated oxide domains generated by local variations in oxide thickness or composition. In contrast, persistent H1 features may reflect interference-related ring-like structures or heterogeneous multilayer oxide distributions associated with structural coloration phenomena.

Fig. 4(a) shows the persistence diagram of β_0 (connected components) for tamahagane. A total of 13,239 features were detected. The points are broadly distributed across the birth–death plane, and notably, several features exhibit birth values below 0.2 yet persist until death values exceeding 0.7, yielding a maximum persistence of 0.71. This indicates that tamahagane possesses color domains that emerge at low hue values and survive over a wide range of the filtration, reflecting spatially coherent color regions of considerable extent.

Fig. 4(b) shows the corresponding β_0 diagram for the converter lance metal, with 12,295 features detected. Although the number of features is comparable to that of tamahagane, the overall scatter of points is more concentrated near the diagonal. The maximum persistence is 0.55, which is noticeably lower than the 0.71 observed for tamahagane. This suggests that while the converter lance metal does contain a similar number of distinct color

domains, those domains are individually less persistent—that is, they span a narrower range of hue values and are spatially less coherent.

Fig. 4(c) presents the β_1 diagram (loops, i.e., enclosed color boundaries) for tamahagane, where 14,657 features were identified. This is the most revealing of the four diagrams. The points are widely scattered above the diagonal, with a mean persistence of 0.116 and a total persistence of 1,707. The presence of numerous high-persistence β_1 features signifies that the tamahagane surface contains many closed loop structures in its color field—regions where one color is topologically enclosed by another. This pattern is consistent with the nested, multi-layered appearance of the structural coloration that has been described as *gōka-kenran*.

Fig. 4(d) shows the β_1 diagram for the converter lance metal, with 13,744 features. In marked contrast to Fig. 4(c), the point cloud is compressed toward the diagonal. The mean persistence is 0.070, which is only 60% of the tamahagane value, and the total persistence is 965—roughly 57% of that for tamahagane. This means that while enclosed color boundaries do exist on the converter lance metal surface, they are both fewer in number at high persistence and individually shorter-lived. In other words, the color structure of the converter lance metal lacks the depth of spatial nesting that characterizes tamahagane.

Taken together, the most striking difference between the two materials lies not in β_0 but in β_1. The number of connected color domains is comparable, yet the way those domains enclose and nest within one another differs substantially. It is this topological nesting—quantified here for the first time through the total and mean persistence of β_1—that appears to underlie the distinction between the *gōka-kenran* of tamahagane and the subdued coloration of the converter lance metal, which Matsui has described as *kanko-soboku* (quiet simplicity).

4. Discussion

4.1 Interpretation of PH Features in Relation to Oxide Multilayer Structures

The PH results can be interpreted not only as image-level color complexity but also as a possible reflection of the underlying oxide-film structure. The fluctuation of Betti numbers observed at specific filtration thresholds may reflect the heterogeneous multilayer structure of the oxide film formed on the Tamahagane surface. As the filtration parameter increases, spatially separated color domains associated with variations in oxide thickness and local refractive index gradually merge, resulting in changes in H0 connectivity. Simultaneously, interference-related ring-like or nonuniform color distributions may generate transient H1 features corresponding to topological loops within the structural color pattern.

The most important finding of the present study is that a decisive difference was observed in the β_1 features (loop structures, i.e., enclosed color boundaries) of the hue channel between the two materials.

The total persistence of β_1 in the hue channel for tamahagane was 1.77 times that of the converter lance metal, and the mean persistence was 1.66 times greater. What this tells us is straightforward: the surface of tamahagane contains far more loop structures—color domains that enclose one another in a nested fashion—than does the converter lance metal, and these structures are individually longer-lived across the filtration. In short, the spatial complexity of the color field on tamahagane is not merely richer in variety but deeper in its topological layering.

We consider this result to be interpretable as a "topological fingerprint" of the multi-layered structural color arising from the three-layer oxide film described by Matsui—specifically, the cyclic sequence of oxidation (Region I), reduction (Region II), and re-oxidation (Region III). The nested oxide structure at the nanoscale manifests, at the optical scale, as nested color boundaries, and it is precisely this nesting that persistent homology captures through β_1.

A notable difference was also found in the β_0 features of the saturation channel. The mean persistence for tamahagane was 2.08 times that of the converter lance metal, and the maximum persistence was 2.30 times greater. This indicates that regions of high saturation on the tamahagane surface are both more prominent and more spatially separated from their surroundings. In terms of the aesthetic vocabulary used by Matsui, this corresponds to a quantification of *gōka-kenran* from the perspective of chromatic vividness, as distinct from the hue-based nesting discussed above.

On the basis of these results, we believe that the present study has demonstrated the possibility of expressing the *gōka-kenran* quality of tamahagane in quantitative, reproducible terms through persistent homology. More concretely, we propose that a "structural color complexity metric" can be constructed by combining two indicators: the total persistence of β_1 in the hue channel and the mean persistence of β_0 in the saturation channel. To the best of

our knowledge, this represents the first attempt to translate the qualitative contrast between *gōka-kenran* and *kanko-soboku*—as described in Matsui's work—into a pair of numerical descriptors that can be independently reproduced by any researcher with access to the same optical micrographs.

We are aware that the present analysis is a preliminary step. The images were resized to 512 × 384 pixels, the hue circularity was handled by linear approximation, and no masking of low-brightness regions was applied. Each of these points leaves room for refinement. In future work, we intend to carry out extended analyses on our local computing environment as well as on cloud-based platforms, varying the image resolution and extracting specific regions of interest (ROI) for focused analysis. Such refinements should yield more convincing and detailed characterization of the topological structure of tamahagane's coloration. We hope that this direction of work will contribute, however modestly, to a deeper understanding of the material that has supported the art of the Japanese sword for centuries.

4.2 Limitations and Future Work

Nevertheless, the dominant persistence structures observed in this study were mainly associated with spatially extensive color domains rather than isolated dark pixels. Therefore, the overall topological tendencies and comparative relationships between samples are considered to remain qualitatively meaningful despite the absence of masking. Future studies will incorporate masking procedures and perceptually uniform color spaces such as CIELAB to improve the robustness of the analysis.

5. Conclusions

In the present study, a comparative investigation was carried out, using persistent homology analysis, between tamahagane—a high-grade steel produced by the traditional *tatara* ironmaking process—and colored regions resembling tamahagane that were formed on lance metal during converter blowing, which constitutes the core operation of modern steelmaking. The following results were obtained.

(1) The β_1 features (enclosed color boundaries) in the hue channel revealed a decisive difference between the two materials. The total persistence for tamahagane was 1.77 times that of the converter lance metal, and the mean persistence was 1.66 times greater. This indicates that the color field of tamahagane possesses a significantly deeper nested structure—color domains enclosed within one another—than that of the converter lance metal.

(2) In the β_0 features (connected components) of the saturation channel, tamahagane exhibited a mean persistence 2.08 times and a maximum persistence 2.30 times greater than those of the converter lance metal. This demonstrates that highly saturated color regions on the tamahagane surface are more vivid and more distinctly separated from their surroundings.

(3) Combining the β_1 total persistence of the hue channel with the β_0 mean persistence of the saturation channel, a "structural color complexity metric" was proposed. This metric provides, to the best of our knowledge, the first quantitative and reproducible expression of the contrast between *gōka-kenran* (gorgeous splendor) and *kanko-soboku* (quiet simplicity) as described by Matsui.

(4) The topological nesting captured by β_1 is consistent with the three-layer oxide film structure (oxidation–reduction–re-oxidation) elucidated by Matsui through process engineering analysis, suggesting that the persistent homology results serve as a topological fingerprint of the underlying metallurgical reaction network.

On the basis of these findings, it is considered that the applicability of persistent homology to the field of materials archaeology has been demonstrated. Furthermore, the present results suggest that this method holds potential as a tool for the quantitative evaluation of aesthetic qualities in works of art and craftsmanship.

Beyond the present analysis of Tamahagane structural colors, the proposed framework suggests broader possibilities for intelligent aesthetic evaluation and cultural heritage engineering. Persistent homology may provide reproducible topological signatures suitable for the digital archiving of traditional metallic materials and craftsmanship. Furthermore, integration with machine learning techniques may enable AI-assisted classification of oxidation states and aesthetic surface features. In future industrial applications, real-time monitoring of structural color evolution during oxidation processes may also become feasible. The present work therefore represents an initial step toward a broader framework of “topological materials science” connecting materials engineering, computational topology, and human aesthetic perception.

Declaration of Generative AI Use

In the preparation of this work, generative AI tools were used for two specific purposes. First, AI-based image stylization was employed to produce schematic representations of the original micrographs (Figs. 1 and 2) in order to avoid copyright infringement, as the original photographs are the property of the Iron and Steel Institute of Japan. Second, since all of the authors are native speakers of Japanese, AI-assisted language tools were used to check and polish the English expressions for accuracy and clarity. The authors have reviewed and take full responsibility for the content of the published article.

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Not applicable.

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